Human HET: *Challenge 1 Segmented Mirror*

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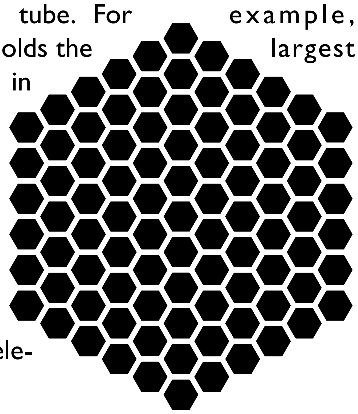
Introduction

The astronomer's quest to see fainter objects and farther into the sky leads to the desire for larger telescopes. But as the size of the telescope gets bigger, so does the price tag. Creative engineers and astronomers use new technology, tools, and fabrication methods to reduce the cost of building large telescopes with large mirrors. For the Hobby-Eberly Telescope (HET), 91 small hexagonal mirrors together act as a single 10 by 11 meter hexagonal mirror. The cost of this array of mirrors is significantly less than a single monster mirror, even when the complex controls needed to keep it acting as one mirror and the structure are taken into account. This design departs radically from that of the other telescopes at McDonald Observatory near Fort Davis, Texas. Each of those telescopes contains a single primary mirror mounted at the bottom end of a telescope tube. For example, the Harlan J. Smith telescope holds the largest primary mirror at 2.7 meters in diameter. Were it rebuilt today, this telescope and dome would cost almost $22 million. The HET cost $13.5 million and gathers 14 times more light with its segmented mirror 10 meters wide. HET has the lowest cost of any telescope in its size class.



Goal

Students discover through problem-based learning the cost-effective nature of HET's mirror array. By solving problems, the students exercise science process skills, apply mathematical knowledge, and test solutions.

Materials

1. Circular paper cutouts, to represent various mirror diameters. Alternatively, use round plastic lids, CDs, pennies, buttons, and so forth as model mirrors. The provided mirror cutout sheet contains a collection of four 100:1 scale mirrors of diameters 1, 2.7, 5, and 10 centimeters. The largest represents a mirror 10 meters wide; the 2.7-centimeter model represents the Harlan J. Smith 2.7-meter telescope mirror. The students can use these models to help visualize a solution.
2. Paper, pencil, ruler with metric scale for each group.
3. Graph of astronomical mirror cost versus diameter.

Texas Essential Knowledge and Skills

Science TEKS Process Skills

- plan and implement investigative procedures (6.2, 7.2, 8.2, IPC, Physics, Astronomy)
- relationships between science and technology (8.5)

Science TEKS Concepts

- Systems (6.5, 7.5)

Math TEKS

- Geometry and spatial reasoning (6.6, 6.7, 7.8)
- Measurement (6.8, 7.9, 8.8, 8.10)
- Underlying processes and mathematical tools (6.11, 7.13)
- Patterns, relationships, and algebraic thinking (7.4, 7.5, 8.5, 8.7,)

Preparation

Divide the students into work groups and pass out materials.

Problem

As part of the HET design team, the students' job is to develop a large and cost-effective primary mirror. It must have a reflective surface area equal to a 10-meter-diameter mirror, and cost as little as possible. Put the students in the role of an astronomer or engineer preparing to solve this problem. Explain that the goal is to build a 10-meter equivalent diameter mirror without spending the cost of a single 10-meter mirror. For instance, if the cost of a 10-meter mirror is $50 million, you might want to say, "Your job is to build a 10-meter equivalent mirror for less than $50 million." Pass out the model mirror cutout sheet. The students should cut out the mirror disks and experiment to see how many mirrors of what sizes are needed to make an area equivalent to a single 10-meter-diameter mirror.

Solution

The key to the solution is to assemble several inexpensive small mirrors, or mirror segments, of equal size into an array that works as one giant 10-meter-diameter mirror. The area sum of these mirror segments is equal to the area of the single 10-meter-diameter mirror. To simplify the problem, use circular mirrors instead of hexagons so that the area formula is

$$\text{Area of a circle} = \pi \times \text{radius}^2 = \pi \times \frac{\text{Diameter}^2}{4}$$

The next step for the students is to figure out how to calculate the total cost, which is the number of mirror segments times the cost per mirror segment. The number of mirrors is the 10-meter mirror area divided by the segment mirror area. The graph shows the cost per mirror versus mirror diameter.

Total Cost = number of mirrors × cost per mirror d: diameter of segment mirror

D: diameter of giant mirror = 10 meters

C: cost per mirror of diameter, d

p: Pi, the circumference to diameter ratio of a circle ~ 3.14159…

$$\text{Total Cost} = \frac{\frac{\pi \times D^2}{4}}{\frac{\pi \times d^2}{4}} \times C = \frac{D^2}{d^2} \times C = \frac{100}{d^2} \times C$$

The students should experiment to minimize total cost by plugging in values for the diameter of the segment mirror, d, and its cost, C. After a few trials, the students should discover that using many small mirrors, whose total area equals the area of the giant mirror, minimizes total cost. You may wish to suggest that the students are not limited to the number of mirrors on the cutout sheet — they can "order" more mirrors.

Extension

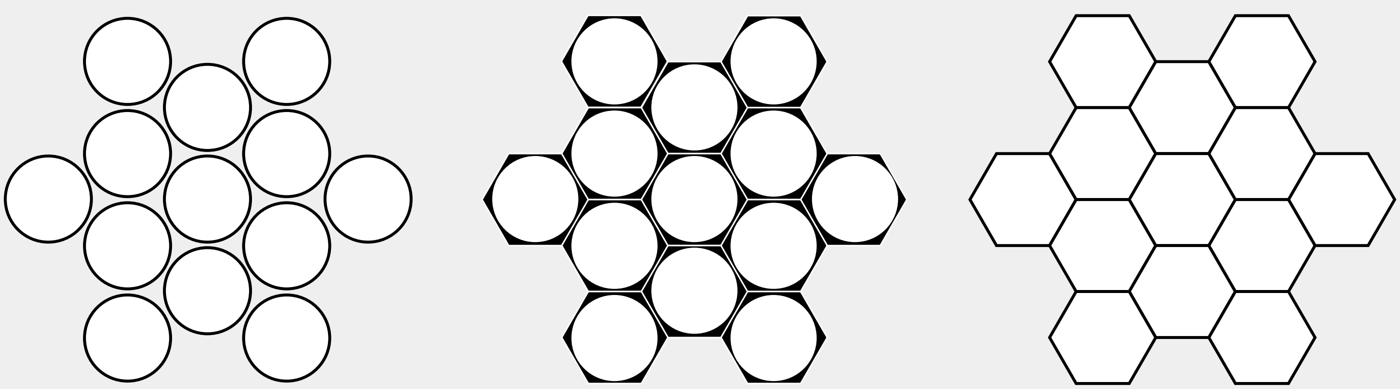
- Challenge the students to determine the dimensions of their segmented, or mirror array, as if they laid out all the round mirror segments with the edges touching neighboring mirrors.
- Ask them why they think HET's mirror segments are hexagons instead of circles.

Conclusion

After the groups have worked out their solutions and cost estimates, invite group representatives to share their results. Explain that this is what scientists do. Scientists regularly meet to share problems and results. They also evaluate and check each other's work. Remind the students that this is not a game show: Multiple solutions exist, and each student has an important contribution to make. Given the circumstances of the problem, one solution may stand out as the best.

Hints

At first, you will spend about 30 minutes establishing the context of this scenario and, most important, engaging the students. Use an inquiry approach to stimulate the students' thinking about what they are about to do. Also keep in mind that telescopes and astronomy may not be familiar subjects for most of the students. If it is not, you may wish to start with a discussion of astronomers and what they do, and then move on to telescopes and observatories. Finally, talk about the costs of telescopes, especially large aperture telescopes that cost upward of $150 million. Students seem to relish the idea that they are in charge of spending money on the order of millions of dollars. This fact helps engage them. Remind them why astronomers need the larger instruments — to gather more light in order to see further and fainter objects. The cost of the primary mirror is a significant part (10% or more) of the total budget for the telescope. As the cost of the primary mirror increases, so does the cost of the telescope. This is not a linear relation, but an exponential relationship. Deciding what size and kinds of mirrors to use is a real-world problem. Astronomers and engineers spent years working out how to build the HET. They had to solve this same problem of building an inexpensive 10-meter class mirror, and built the entire HET observatory for under $15 million.



The hexagons on the right fit together edge to edge; little space is wasted between the segments. Compare that with an array of circles on the left. The gaps show up by superimposing the arrays (center). Even though each array takes up the same amount of space, the hexagonal mirror array reflects more light than the circular mirror array.